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The decorated ferrimagnetic Ising model in a transverse field

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Abstract. The effect of a transverse field on the magnetizations and phase diagrams of a decorated two-sublattice Ising model ferrimagnetic consisting of two magnetic atoms A and B with spins $\sigma_A = 1/2$ and $S_B = 1$ is investigated within the framework of the effective field theory. A number of characteristic phenomena, such as the possibility of compensation points and two transitions, are found.

1. Introduction

Ferrimagnetism has been extensively investigated in the past both experimentally and theoretically, since important magnetic materials for technological applications, such as garnets and ferrites, are ferrimagnetic. Ferrimagnets have several sublattices with a finite resultant moment and show paramagnetic behaviour above the transition temperature T_c. In contrast with a ferromagnet, there is an interesting possibility of the existence, under certain conditions, of a compensation temperature T_k ($T_k < T_c$), at which the resultant magnetization vanishes [1, 2]. In recent works, the effect of disordered interfaces with alloying type A_p B_{p-1} on the transition temperature and magnetization has been investigated for a bilayer system consisting of two magnetic layers A and B where A and B can possess different bulk properties [3–6]. The effects of a crystal field D and the magnitude of the spin S on the phase diagram (T_c and T_k) have been examined and it was clarified that more than one compensation point can exist in the disordered ferrimagnetic alloy as well as the diluted mixed spin-1/2 and spin-1 ferrimagnetic Ising systems [7–11].

Decorated Ising spin models were originally introduced in the literature by Syozi [12] as exactly solvable models in statistical physics. They show several kinds of ferrimagnetic behaviour in the temperature dependence of the resultant magnetization according to the assumed values of parameters. The arrangement of atoms in these models is like that in the normal spinel. However, most of the decorated models studied have been restricted to the effects of a crystal field on the phase diagram [13–16]. On the other hand, there has been considerable interest in the study of quantum fluctuations in classical spin models. The simplest of such systems is the Ising model in a transverse field. The spin-1/2 transverse Ising model, originally introduced by De Gennes [17] as a valuable model for tunnelling of the proton in hydrogen-bonded ferroelectrics [18] such as those of KH_2PO_4 type, has been extensively studied by the use of various techniques [19–23] including the effective theory field treatment [24, 25] based on a generalized but approximated Callen–Suzuki relation derived by Sà Barreto *et al*.

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The purpose of this work is to study via effective field theory [26, 27] the effect of a transverse field on the magnetic properties (the critical temperature T_c, the compensation temperature T_k and the magnetization curves) of a decorated two sublattice ferrimagnetic Ising system consisting of two magnetic atoms A and B with spins $\sigma_A = 1/2$ and $S_B = 1$. As far as we know, such a study has not been carried out. In particular the results obtained here may clarify the fact that the applied transverse field can control the compensation points. Therefore, the outline of this work is as follows. The formulation of the problem is given in section 2 on the basis of the effective field theory. The results are discussed in section 3 and a brief conclusion is given in section 4.

2. Formulation

We consider a decorated ferrimagnetic Ising system. The whole lattice is divided into two sublattices L_1 and L_2. Every point of L_1 is always occupied by an A atom with the fixed spin σ_A ($\sigma_A = 1/2$). That of L_2, which is composed of one decorating point on every bond of L_1, is always occupied by a B atom with a fixed spin S_B ($S_B = 1$). The exchange interaction between A and B atoms is assumed to be antiferromagnetic. Furthermore, we assume that there exists a ferromagnetic exchange interaction between every nearest-neighbour pair of A atoms. For clarification, the two-dimensional decorated system is depicted in figure 1. The Hamiltonian of the system has the form

$$H = J \sum_{(ij)} \sigma_i^z S_j^z - J \sum_{(ij)} \sigma_i^z \sigma_j^z - \Omega_1 \sum_i \sigma_i^x - \Omega_2 \sum_i S_i^x \tag{1}$$

where σ_1^z (S_1^z) and σ_1^x (S_1^x) denote the z and x components of a quantum spin $\vec{\sigma_1^z}$ ($\vec{S_1^z}$) of magnitude $\sigma = 1/2$ ($S = 1$) at site i (m) and Ω_1 (Ω_2) is the transverse field acting on σ_i^z (S_i^z). J and J' ($J > 0$ and $J' < 0$) are the exchange interactions. The first two summations are carried out only over nearest-neighbour pairs of spins.

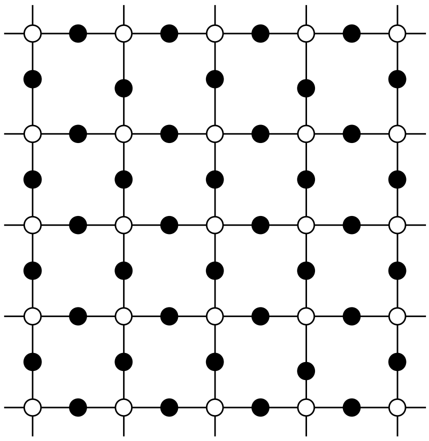

Figure 1. The two-dimensional decorated spin system consisting of magnetic atoms A and B with spin values $\sigma_A = 1/2$ and $\sigma_B = 1$, where the A atoms (white points) form a square lattice.

Using a single-site cluster approximation in which attention is focused on a cluster comprising just a single selected spin labelled 0, and the neighbouring spins with which it directly interacts, the Hamiltonian can split into two parts, $H = H_0 + H'$ where H_0 includes all the parts of H associated with the site 0.

Following Sà Barreto *et al* [24, 25], we can use the approximate relation derived for the transverse Ising model,

$$\langle S_{0\alpha}^p \rangle = \left\langle \frac{\mathrm{tr}_0[S_{0\alpha}^p \ \exp(-\beta H_0)]}{\mathrm{tr}_0[\exp(-\beta H_0)]} \right\rangle \tag{2}$$

that neglects the fact that H_0 and H' do not commute. In the limit $\Omega \to 0$, the Hamiltonian contains only S_z and (2) becomes exact. The single angular brackets denote the thermal average for a fixed spatial configuration of the spins.

For a decorated system the quantities of interest are the longitudinal magnetizations m_z^S and m_z^σ, the transverse magnetizations m_x^S and m_z^σ and the quadrupolar moments q_z^S and q_x^S which are defined by

$$m_\alpha^S = \langle\langle S_{0\alpha}\rangle\rangle_c \tag{3}$$

$$m_\alpha^\sigma = \langle\langle S_{0\alpha}\rangle\rangle_c \tag{4}$$

$$q_\alpha^S = \langle\langle S_{0\alpha}^2\rangle\rangle_c \tag{5}$$

where $\alpha = z, x$ and $\langle \ldots \rangle_c$ indicates the usual canonical ensemble thermal average for a given configuration.

The evaluation of the trace in (2) yields the following equations:

$$m_Z^\sigma = \frac{1}{2}\left\langle \left(\sum_{j=1}^{N}(-JS_j^z) + \sum_{j=1}^{M}(J'\sigma_j^z) \right) \left(\left[\sum_{j=1}^{N}(-JS_j^z) + \sum_{j=1}^{M}(J'\sigma_j^z) \right]^2 + \Omega_1^2 \right)^{-1/2} \times \tanh\left[\frac{1}{2}\beta \sqrt{\left[\sum_{j=1}^{N}(-JS_j^z) + \sum_{j=1}^{M}(J'\sigma_j^z) \right]^2 + \Omega_1^2} \right] \right\rangle \tag{6}$$

$$m_X^\sigma = \frac{1}{2}\left\langle \Omega_1 \left(\left[\sum_{j=1}^{N}(-JS_j^z) + \sum_{j=1}^{M}(J'\sigma_j^z) \right]^2 + \Omega_1^2 \right)^{-1/2} \times \tanh\left[\frac{1}{2}\beta \sqrt{\left[\sum_{j=1}^{N}(-JS_j^z) + \sum_{j=1}^{M}(J'\sigma_j^z) \right]^2 + \Omega_1^2} \right] \right\rangle \tag{7}$$

$$m_Z^S = \left\langle \left(-\sum_{j=1}^{N'} J\sigma_j^z \right) \left(\left(\sum_{j=1}^{N'} J\sigma_j^z \right)^2 + \Omega_2^2 \right)^{-1/2} 2\sinh\left[\beta \sqrt{\left(\sum_{j=1}^{N'} J\sigma_j^z \right)^2 + \Omega_2^2} \right] \times \left(2\cosh\left[\beta \sqrt{\left(\sum_{j=1}^{N'} J\sigma_j^z \right)^2 + \Omega_2^2} \right] + 1 \right)^{-1} \right\rangle \tag{8}$$

$$q_Z^S = \left\langle \left(2\left(\sum_{j=1}^{N'} J\sigma_j^z \right)^2 + \Omega_2^2 \right) \left(\left(\sum_{j=1}^{N'} J\sigma_j^z \right)^2 + \Omega_2^2 \right)^{-1/2} 2\cosh\left[\beta \sqrt{\left(\sum_{j=1}^{N'} J\sigma_j^z \right)^2 + \Omega_2^2} \right] \times \left(2\cosh\left[\beta \sqrt{\left(\sum_{j=1}^{N'} J\sigma_j^z \right)^2 + \Omega_2^2} \right] + 1 \right)^{-1} \right\rangle \tag{9}$$

$$m_X^S = \left\langle \Omega_2 \left(\left(\sum_{j=1}^{N'} J\sigma_j^z \right)^2 + \Omega_2^2 \right)^{-1/2} 2\sinh\left[\beta \sqrt{\left(\sum_{j=1}^{N'} J\sigma_j^z \right)^2 + \Omega_2^2} \right] \times \left(2\cosh\left[\beta \sqrt{\left(\sum_{j=1}^{N'} J\sigma_j^z \right)^2 + \Omega_2^2} \right] + 1 \right)^{-1} \right\rangle \tag{10}$$

$$q_X^S = \left\langle \left(\left(\sum_{j=1}^{N'} J\sigma_j^z \right)^2 + 2\Omega_2^2 \right) \left(\left(\sum_{j=1}^{N'} J\sigma_j^z \right)^2 + \Omega_2^2 \right)^{-1/2} 2\cosh\left[\beta \sqrt{\left(\sum_{j=1}^{N'} J\sigma_j^z \right)^2 + \Omega_2^2} \right] \times \left(2\cosh\left[\beta \sqrt{\left(\sum_{j=1}^{N'} J\sigma_j^z \right)^2 + \Omega_2^2} \right] + 1 \right)^{-1} \right\rangle \tag{11}$$

where N and M (N') are the numbers of nearest neighbours of central site i (m); we are going to work on a bidimensional system ($N = M = 4$ and $N' = 2$). m_α^σ (m_α^S) with $\alpha = z$ or x are the longitudinal and transverse magnetizations corresponding to $\sigma = 1/2$ ($S = 1$); q_α^S ($\alpha = z, x$) are the longitudinal and transverse quadrupolar moments corresponding to $S = 1$. $\beta = 1/k_B T$ (we take $k_B = 1$ for simplicity), $\langle \ldots \rangle$ indicates the usual canonical ensemble thermal average for a given configuration and the sums run over all nearest neighbours.

To perform thermal averaging on the right-hand side of equations (6)–(11), we follow the general approach described in [15] and [16]. First of all, in the spirit of effective field theory, multi-spin correlation functions are approximated by products of single-spin averages. We then take advantage of the integral representation of the Dirac delta distribution, in order to write equations (6), (7), (8), (9) and (10), (11) in the form

$$m_\alpha^\sigma = \int \mathrm{d}w\, f_\alpha^\sigma(w, \Omega_1) \frac{1}{2\pi} \int \mathrm{d}\lambda\, \mathrm{e}^{\mathrm{i}w\lambda} \prod_{m=1}^{N} \langle \mathrm{e}^{\mathrm{i}\lambda J S_m^z} \rangle \prod_{j=1}^{M} \langle \mathrm{e}^{\mathrm{i}\lambda J' \sigma_j^z} \rangle \tag{12}$$

$$m_\alpha^S = \int \mathrm{d}w\, f_\alpha^S(w, \Omega_2) \frac{1}{2\pi} \int \mathrm{d}\lambda\, \mathrm{e}^{\mathrm{i}w\lambda} \prod_{j=1}^{N'} \langle \mathrm{e}^{\mathrm{i}\lambda J \sigma_j^z} \rangle \tag{13}$$

$$q_\alpha^S = \int \mathrm{d}w\, g_\alpha^S(w, \Omega_2) \frac{1}{2\pi} \int \mathrm{d}\lambda\, \mathrm{e}^{\mathrm{i}w\lambda} \prod_{j=1}^{N'} \langle \mathrm{e}^{\mathrm{i}\lambda J \sigma_j^z} \rangle \tag{14}$$

where

$$f_z^\sigma(y_1, \Omega_1) = \frac{1}{2} \frac{y_1}{\sqrt{y_1^2 + \Omega_1^2}} \tanh[\tfrac{1}{2}\beta\sqrt{y_1^2 + \Omega_1^2}] \tag{15}$$

$$f_z^S(y_2, \Omega_2) = \frac{y_2}{\sqrt{y_2^2 + \Omega_2^2}} \frac{2\sinh[\beta\sqrt{y_2^2 + \Omega_2^2}]}{2\cosh[\beta\sqrt{y_2^2 + \Omega_2^2}] + 1} \tag{16}$$

$$g_z^S(y_2, \Omega_2) = \frac{1}{y_2^2 + \Omega_2^2} \frac{1(2y_2^2 + \Omega_2^2)\cosh[\beta\sqrt{y_2^2 + \Omega_2^2}] + \Omega_2^2}{2\cosh[\beta\sqrt{y_2^2 + \Omega_2^2}] + 1} \tag{17}$$

with

$$f_x^\sigma(y_1, \Omega_1) = f_z^\sigma(\Omega_1, y_1) \qquad f_x^S(y_2, \Omega_2) = f_z^S(\Omega_2, y_2) \qquad g_x^S(y_2, \Omega_2) = g_z^S(\Omega_2, y_2).$$

We now introduce the probability distribution of the spin variables (for details see Saber [26] and Tucker *et al* [27]).

$$P(\sigma^z) = \tfrac{1}{2}[(1 + 2m_z^\sigma)\delta(\sigma^z - \tfrac{1}{2}) + (1 - 2m_z^\sigma)\delta(\sigma^z + \tfrac{1}{2})] \tag{18}$$

$$P(S^z) = \tfrac{1}{2}(q_z^S + m_z^S)\delta(S^z - 1) + (1 - q_z^S)\delta(S^z) + \tfrac{1}{2}(q_z^S - m_z^S)\delta(S^z + 1). \tag{19}$$

Using these expressions and equations (12)–(14), we obtain the following set of equations:

$$m_z^\sigma = \frac{1}{2^{M+N}} \sum_{\mu=0}^{M} \sum_{\nu=0}^{N} \sum_{\gamma=0}^{N-\nu} \sum_{i_1=0}^{\mu} \sum_{i_2=0}^{M-\mu} \sum_{j_1=0}^{\nu} \sum_{j_2=0}^{\gamma} \sum_{j_3=0}^{N-(\nu+\gamma)} C_\mu^M C_\nu^N C_\gamma^{N-\nu} C_{i_1}^\mu C_{i_2}^{M-\mu} C_{j_1}^\nu C_{j_2}^\gamma C_{j_3}^{N-(\nu+\gamma)}$$
$$\times 2^{\nu+i_1+i_2} (-1)^{\nu+i_1+i_2} (m_z^\sigma)^{i_1+i_2} (q_z^S)^{j_1+j_2+j_3} (m_z^S)^{N-(\nu+j_2+j_3)}$$
$$\times f_z^\sigma \left(\frac{J'}{2}(M-2\mu) - J(N-(\nu+2\gamma), \Omega_1) \right) \tag{20}$$

$$m_z^S = \frac{1}{2^{N'}} \sum_{\mu=0}^{N'} \sum_{k_1=0}^{\mu} \sum_{k_2=0}^{N'-\mu} C_\mu^{N'} C_{k_1}^\mu C_{k_2}^{N'-\mu} (-1)^{k_1} 2^{k_1+k_2} (m_z^\sigma)^{k_1+k_2} f_z^S \left(\frac{-J}{2}(N'-2\mu), \Omega_2 \right) \tag{21}$$

$$q_z^S = \frac{1}{2^{N'}} \sum_{\mu=0}^{N'} \sum_{k_1=0}^{\mu} \sum_{k_2=0}^{N'-\mu} C_\mu^{N'} C_{k_1}^\mu C_{k_2}^{N'-\mu} (-1)^{k_1} 2^{k_1+k_2} (m_z^\sigma)^{k_1+k_2} g_z^S \left(\frac{-J}{2}(N'-2\mu), \Omega_2 \right). \tag{22}$$

We have thus obtained a set of self-consistent equations (20)–(22) for the moments that can be solved directly by numerical iterations. The total longitudinal magnetization M_z of the system is given by

$$\frac{M_z}{N_A} = m_z^\sigma + 2m_z^S \tag{23}$$

where N_A is the total number of A atoms. The sublattice longitudinal magnetization m_z^S can be evaluated as

$$m_z^S = -2m_z^\sigma f_z^S(J, \Omega_2). \tag{24}$$

Thus, the total longitudinal magnetization M_z can be expressed as

$$M_z = M_z^0[1 - 4f_z^S(J, \Omega_2)] \tag{25}$$

where

$$M_z^0 = N_A m_z^\sigma.$$

Below the transition temperature T_c, M_z^0 takes a finite value. Accordingly, if the compensation point at which the total magnetization reduces to zero may exist in the system, the compensation temperature T_k can be determined exactly from the condition $M_z = 0$ for $T_k < T_c$, namely

$$1 = 4f_z^S(J, \Omega_2) \qquad \text{for } T_k < T_c. \tag{26}$$

In the vicinity of the transition temperature T_c, equation (18) can be expressed as

$$q = q_0 + q_1\sigma^2 \tag{27}$$

where

$$q_i = \frac{1}{2^{N'}} \sum_{\mu=0}^{N'} \sum_{k_1=0}^{\mu} \sum_{k_2=0}^{N'-\mu} C_\mu^{N'} C_{k_1}^\mu C_{k_2}^{N'-\mu} (-1)^{k_1} 2^{k_1+k_2} g_z^S \left[\frac{-J}{2}(N'-2\mu), \Omega_2 \right] \delta[k_1+k_2, i].$$

In order to determine the transition temperature T_c, let us expand the right-hand sides of equations (20) and (21); using equation (22), we obtain the following equation for σ and m:

$$\sigma = a_{10}\sigma + a_{01}m + a_{21}\sigma^2 m + a_{12}\sigma m^2 + a_{30}\sigma^3 + a_{03}m^3 + \cdots \tag{28}$$
$$m = b_{10}\sigma \tag{29}$$

where the coefficients a_{ij} are given by

$$a_{ij} = \frac{1}{2^{M+N}} \sum_{\mu=0}^{M} \sum_{\nu=0}^{N} \sum_{\gamma=0}^{N-\nu} \sum_{i_1=0}^{\mu} \sum_{i_2=0}^{M-\mu} \sum_{j_1=0}^{\nu} \sum_{j_2=0}^{\gamma} \sum_{j_3=0}^{N-(\nu+\gamma)} \sum_{i_3}^{j_1+j_2+j_3} C_\mu^M C_\nu^N \times C_\gamma^{N-\nu} C_{i_1}^{\mu} C_{i_2}^{M-\mu} C_{j_1}^{\nu} C_{j_2}^{\gamma} C_{j_3}^{N-(\nu+\gamma)} C_{i_3}^{j_1+j_2+j_3} 2^{\nu+i_1+i_2} (-1)^{\nu+i_1+i_2} q_0^{j_1+j_2+j_3-i_3} q_2^{i_3} \times f_z^\sigma \left[\left(\frac{J'}{2}(M-2\mu) - J(N-(\nu+2\gamma) \right), \Omega_1 \right] \delta[i_1+i_2+2i_3, i] \times \delta[(N-(\nu+j_2+j_3)), j]$$

and

$$b_{10} = \frac{1}{2^{N'}} \sum_{\mu=0}^{N'} \sum_{k_1=0}^{\mu} \sum_{k_2=0}^{N'-\mu} C_\mu^{N'} C_{k_1}^{\mu} C_{k_2}^{N'-\mu} (-1)^{k_1} 2^{k_1+k_2} f_z^S \left[\frac{-J}{2}(N'-2\mu), \Omega_2 \right] \delta[k_1+k_2, 1].$$

If we substitute equation (25) into equation (24), we obtain an equation for σ of the form

$$\sigma = a\sigma + b\sigma^3 + \cdots \tag{30}$$

with

$$a = a_{10} + a_{01}b_{10} \qquad \text{and} \qquad b = a_{12}b_{10}^2 + a_{21}b_{10} + a_{12}b_{10}^2 + a_{30} + a_{03}b_{10}^3.$$

The second-order phase transition line is then determined by the condition

$$a = 1 \qquad \text{and} \qquad b < 0. \tag{31}$$

The sublattice magnetization σ is given by

$$\sigma^2 = \frac{1-a}{b}. \tag{32}$$

For the transition to be of the second order, the right-hand side of equation (32) must be positive. If this is not the case, the transition is of first order, and hence the point at which $a = 1$ and $b = 0$ is the tricritical point. To find the first-order transition line in the $(T_C/J, \Omega/J)$ plane we proceed as follows: apply an external magnetic field h/J and derive the equations analogous to (20)–(22) to obtain the magnetization m_z^σ as a function of T/J, α ($\alpha = J'/J$), Ω/J ($\Omega_1 = \Omega_2 = \Omega$) and h/J. If the transition is of first order the isotherms in the (m_z^σ, h/J) plane, for fixed values of T/J, Ω/J and α have the typical S-shape of the Van der Waals isotherms and, as usual, the first-order transition point is determined by the Maxwell rule. We extrapolate to $h/J = 0$ to obtain the first-order transition temperature when no external field is applied as a function of Ω/J and α.

3. Results and discussion

We are now able to study the magnetic properties (compensation and transition temperature and magnetization curves) of the two-dimensional decorated ferrimagnetic Ising model in a transverse field. For simplicity, we assume that the transverse field acting on the system is homogeneous, i.e. $\Omega_1 = \Omega_2 = \Omega$.

For the system under consideration, the calculations show that the right-hand side of equation (32) is always positive. Hence, all the transitions are of second order.

First, let us examine the variation of the transition temperature versus the magnitude of the transverse field Ω/J for several values of the ratio α of the exchange interactions J' and J ($\alpha = J'/J$). The results are shown in figure 2. The dotted line denotes the compensation temperature T_k/J which is obtained from equation (22), while the solid lines denote the critical

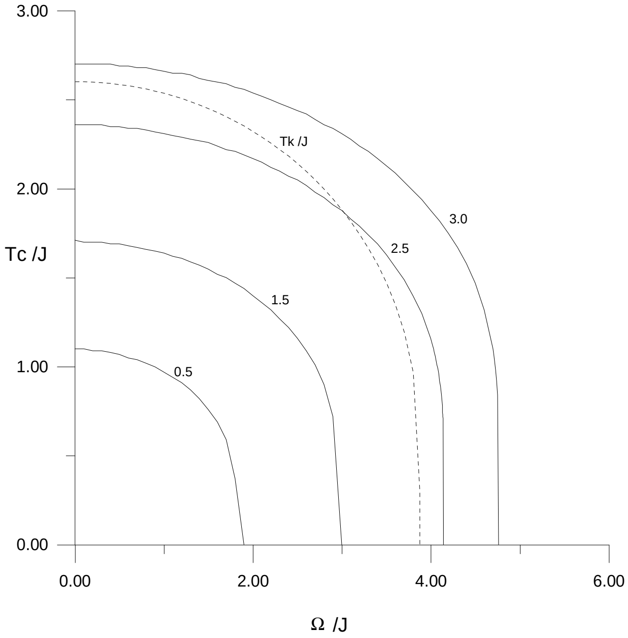

Figure 2. The phase diagram in the $(T/J,\ \Omega/J)$ plane of the decorated ferrimagnetic system depicted in figure 1. The solid and dashed lines represent the critical temperature T_c and the compensation temperature T_k respectively. The number labelling each line denotes the parameter α.

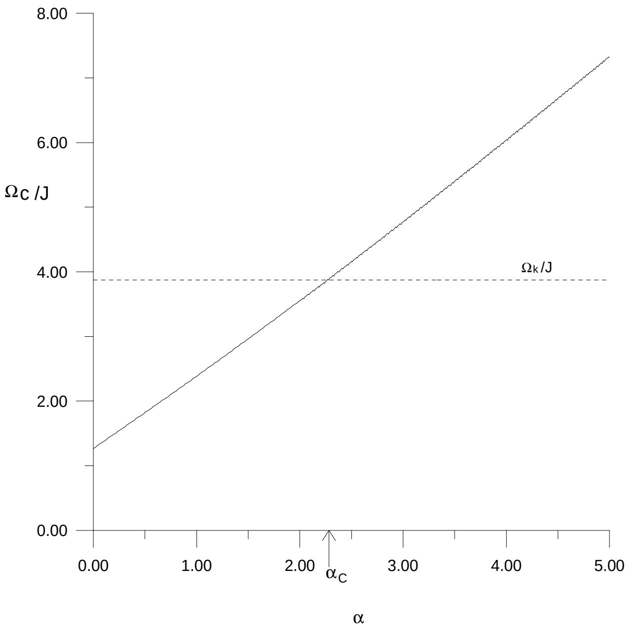

Figure 3. The variation of the critical transverse field Ω_c/J with α (solid line); the dashed line represents the compensation transverse field Ω_k/J. α_c is the value of α above which the system exhibits a compensation point.

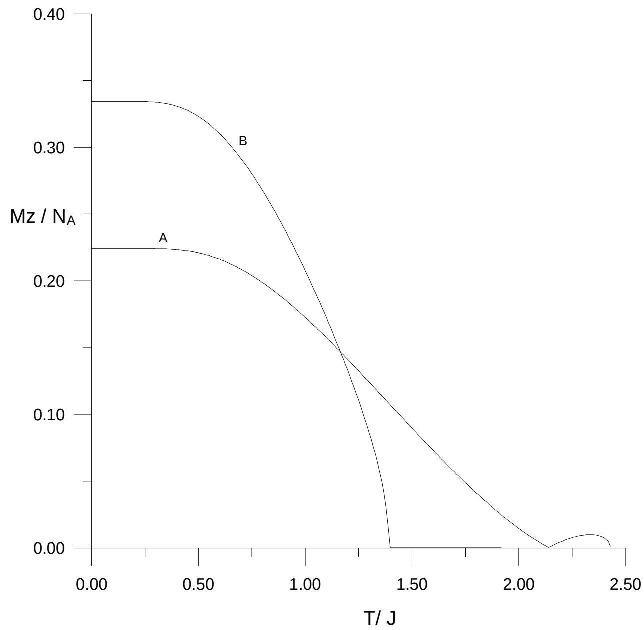

Figure 4. The temperature dependence of the total magnetizations M_z is plotted for the two-dimensional decorated ferrimagnetic system, when two special sets of pair values $(\Omega/J, \alpha)$ are selected: the curve labelled A is obtained for (2.5,3.0); the curve labelled B is the result for (2.0, 1.5).

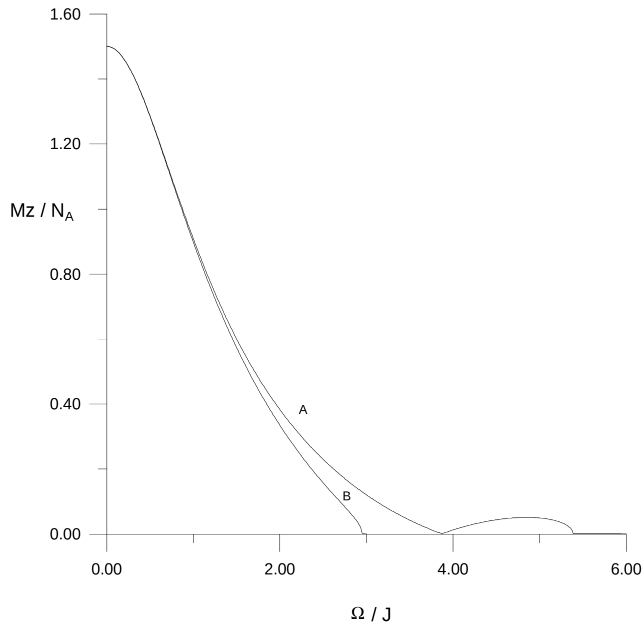

Figure 5. The variation of the total magnetization M_z with Ω for $T = 0$, for selected values of α. The curve A corresponds to $\alpha = 3.5$ and the curve B corresponds to $\alpha = 1.5$.

temperature T_c/J (which is obtained from equations (27)).We can see that T_k/J is independent of α. It is shown that both T_k/J and T_c/J decrease from their maximum values which correspond to $\Omega/J = 0$ to vanish at some critical value of the transverse field. Depending on the values of α and Ω, T_k/J is smaller or larger than T_c/J, that is the compensation point does not exist for any values of α and Ω. We have found that for $\alpha > 2.88$ there is a compensation point for any Ω smaller than Ω_c (see curve $\alpha = 3.0$), for $2.30 < \alpha < 2.88$ we have a compensation point only in a limited range of Ω: for example, $\alpha = 2.5$, the system exhibit a compensation point for $3.873 < \Omega < 4.142$, and for $\alpha < 2.30$ there is no compensation phenomena.

In figure 3 we have plotted the variation, with α, of the critical transverse field Ω_c/J (compensation transverse field Ω_k/J) at which the critical temperature (compensation temperature) reduces to zero. It is seen that Ω_c/J varies linearly with α, while Ω_k/J remains constant and is equal to 3.873. A compensation point exist only if $\Omega_c/J > \Omega_k/J$, that is when α is greater than a critical value which is equal to 2.30.

Let us now examine the variations of the total magnetization M_z of the system. Figure 4 shows the variations of M_z with the temperature for selected values of Ω and α. It is seen that the M_z curve corresponding to $\alpha = 3.0$ and $\Omega = 1.0$ exhibits a compensation point below T_c, while the M_z curve corresponding to $\alpha = 1.5$ and $\Omega = 2.0$ does not show any compensation point. The variations of M_z with Ω when $T = 0$ are shown in figure 5. Depending on α, M_z can have two different behaviours. For $\alpha > \alpha_c$, M_z shows a compensation point (curve A) and for $\alpha < \alpha_c$ there is no compensation point (curve B). All of these results are consistent with the predictions derived from figures 1 and 2.

4. Conclusion

In this work we have investigated the magnetic properties of a two-sublattice decorated Ising ferrimagnetic system composed of two magnetic atoms A and B with $S_A = 1/2$ and $S_B = 1$ in a transverse field. We have shown that the compensation temperature T_k has the same behaviour as the critical temperature T_c; that is T_k decreases when we increase the transverse field Ω and vanishes at a critical value of Ω which is the compensation transverse field Ω_k. The compensation temperature and transverse field (T_k, Ω_k) are independent of the ratio α. The system studied here may be simple, but fruitful, from both the theoretical and material science points of view. We hope that our study will stimulate further the theoretical investigations and/or experimental measurements

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